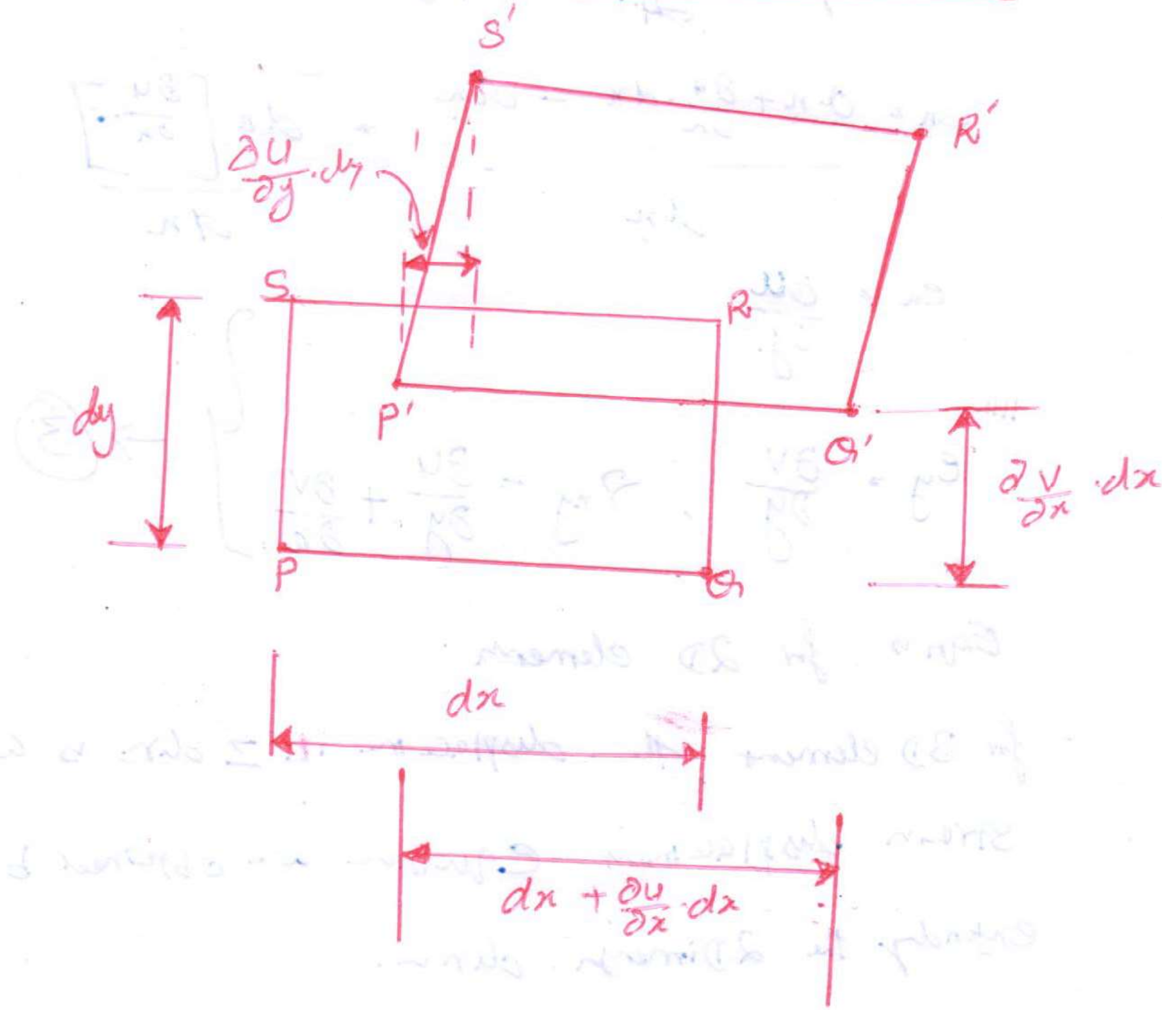
Strain Displacement Relationship Equation.



* PQRS is 2D element. When external force acts on the element, it undergoes deformation and it becomes P'Q'R'S'. Displacement in the x direction is u and y direction is V.

$$\text{Strain } e_x = \frac{P'Q' - PQ}{PQ} \rightarrow (1) \quad \text{when } PQ = dx$$

$$(P'Q')^2 = \left[dx + \frac{\partial u}{\partial x}.dx\right]^2 + \left[\frac{\partial V}{\partial x}.dx\right]^2$$

Calculate P'Q' using the binomial theorem and neglecting the higher order elements (or) $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial x}\right)^2$

$$P'Q' = dx + \frac{\partial u}{\partial x} \cdot dx \rightarrow (2).$$

Substituting equation ~~Eq.~~ (2) in (1)

$$e_x = \frac{dx + \frac{\partial u}{\partial x} \cdot dx - dx}{dx} = \frac{dx \left[\frac{\partial u}{\partial x} \cdot c \right]}{dx}$$

$$\left.\begin{array}{l} e_x = \dfrac{\partial u}{\partial y}. \\ \text{|||} \\ e_y = \dfrac{\partial v}{\partial y}, \quad \gamma_{xy} = \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}. \end{array}\right\} \rightarrow (3)$$

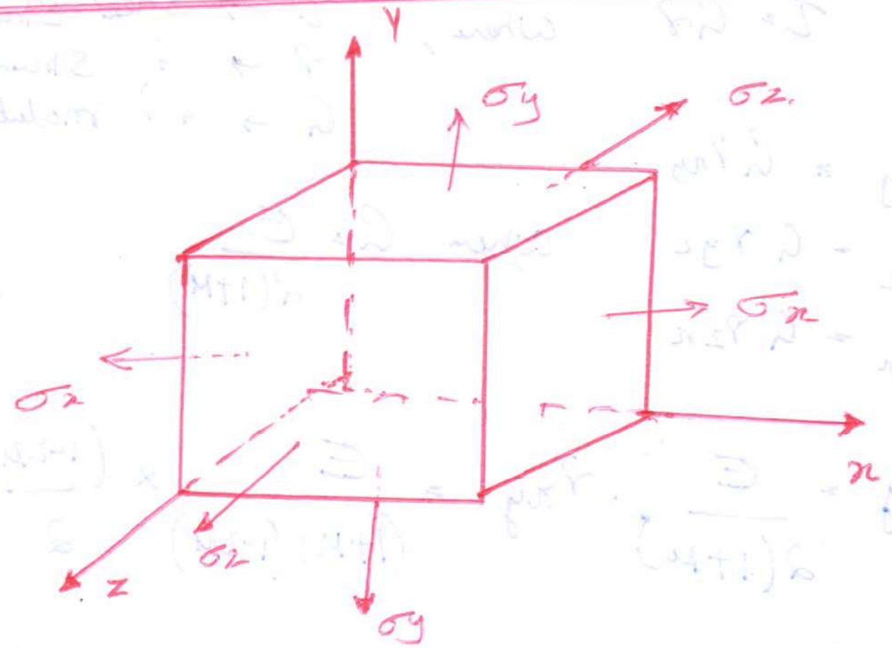
Eqn is for 2D element.

for 3D element, the displacement in Z dir is w.

Strain displacement equation are obtained by extending the 2 Dimension direction.

$$\left.\begin{array}{l} e_z = \dfrac{\partial w}{\partial z}. \\ \gamma_{xz} = \dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x} \\ \gamma_{yz} = \dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial y} \end{array}\right\} \rightarrow (4)$$

Stress Strain Relationship or Constitution matrix [D] for 2D Element. (12)



$$e_x = \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} - \mu \cdot \frac{\sigma_z}{E}$$

$$e_y = \frac{\sigma_y}{E} - \mu \frac{\sigma_x}{E} - \mu \frac{\sigma_z}{E}$$

$$e_z = \frac{\sigma_z}{E} - \mu \cdot \frac{\sigma_x}{E} - \mu \cdot \frac{\sigma_y}{E}$$

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$$\left.\begin{aligned}
\sigma_x &= \frac{E}{(1+\mu)(1-2\mu)}\left[e_x(1-\mu) + \mu \cdot e_y + \mu \cdot e_z\right] \\
\sigma_y &= \frac{E}{(1+\mu)(1-2\mu)}\left[\mu \cdot e_x + e_y(1-\mu) + \mu e_z\right] \\
\sigma_z &= \frac{E}{(1+\mu)(1-2\mu)}\left[\mu \cdot e_x + \mu \cdot e_y + (1-\mu) e_z\right]
\end{aligned}\right\} \rightarrow (1)$$

Shear Stress and Shear strain relation Shear is given by

$\tau = G\gamma$ where, $\tau \rightarrow$ Shear Stress
$\gamma \rightarrow$ " Strain
$G \rightarrow$ " modulus.

$\tau_{xy} = G\gamma_{xy}$

$\tau_{yz} = G\gamma_{yz}$ where $G = \dfrac{E}{2(1+\mu)}$

$\tau_{zx} = G\gamma_{zx}$

$$\tau_{xy} = \frac{E}{2(1+\mu)} \cdot \gamma_{xy} = \frac{E}{(1+\mu)(1-2\mu)} \times \frac{(1-2\mu)}{2} \gamma_{xy}$$

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$$\tau_{yz} = \frac{E}{(1+\mu)(1-2\mu)} \times \frac{(1-2\mu)}{2} \cdot \gamma_{xy} \quad \rightarrow (2)$$

$$\tau_{zx} = \frac{E}{(1+\mu)(1-2\mu)} \times \left(\frac{1-2\mu}{2}\right) \gamma_{zx}.$$

Subs eqn (1) & (2) in matrix form.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \frac{E}{(1+\mu)(1-2\mu)} \begin{bmatrix} (1-\mu) & \mu & \mu & 0 & 0 & 0 \\ \mu & (1-\mu) & \mu & 0 & 0 & 0 \\ \mu & \mu & (1-\mu) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\mu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\mu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\mu}{2} \end{bmatrix} \begin{Bmatrix} e_x \\ e_y \\ e_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}$$

$$\sigma = [D]\{e\}$$

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$$[D] = \frac{E}{(1+\mu)(1-2\mu)} \begin{bmatrix} (1-\mu) & \mu & \mu & 0 & 0 & 0 \\ \mu & (1-\mu) & \mu & 0 & 0 & 0 \\ \mu & \mu & (1-\mu) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\mu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\mu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\mu}{2} \end{bmatrix} \rightarrow I$$

Plane stress:

for 2D elem $\tau_z = 0$ $\gamma_{xz} = \gamma_{yz} = 0 = \sigma_z$

$$\left.\begin{aligned} e_x &= \frac{\sigma_x}{E} - \mu \cdot \frac{\sigma_y}{E} \\ e_y &= \frac{\sigma_y}{E} - \mu \cdot \frac{\sigma_x}{E} \end{aligned}\right\} \rightarrow 1$$

resolv both eqn of 1. we get

$$e_x \Rightarrow \frac{\sigma_x}{E} - \cancel{\mu \cdot \frac{\sigma_y}{E}}$$

$$e_y \times \mu \Rightarrow \cancel{\frac{\sigma_y}{E}\mu} - \mu^2 \frac{\sigma_x}{E}$$

$$e_x + \mu \cdot e_y = \frac{\sigma_x}{E} - \mu^2 \cdot \frac{\sigma_x}{E}$$

$$\mu \cdot e_y + e_x = \frac{\sigma_x}{E} - \mu^2 \cdot \frac{\sigma_x}{E} \cancel{-\mu \cdot e_y}$$

$$\text{"} = \frac{\sigma_x}{E}(1-\mu^2) \Rightarrow \sigma_x = \frac{E}{(1-\mu^2)}(e_x + \mu \cdot e_y) \rightarrow 2$$

Mly we get

$$\sigma_y = \frac{E}{(1-\mu^2)}(\mu \cdot e_x + e_y) \rightarrow 3$$

$$\tau_{xy} = G \cdot \gamma_{xy}$$

$$= \frac{E}{2(1+\mu)} \cdot \gamma_{xy}$$

$$\tau_{xy} = \frac{E}{(1+\mu)(1-\mu)} \times \frac{(1-\mu)}{2} \cdot \gamma_{xy} \rightarrow (4)$$

arrange (2), (3) & (4) in matrix form

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1-\mu^2)} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix} \begin{Bmatrix} e_x \\ e_y \\ \gamma_{xy} \end{Bmatrix}.$$

$$\sigma = [D]\{e\}$$

$$[D] = \frac{E}{(1-\mu^2)} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix}$$

Plane Strain.

For plane strain, we assume the following strains to be zero, $e_z = \gamma_{xz} = \gamma_{yz} = 0$.

shear stress. $\tau_{xz} = \tau_{yz} = 0$, but $\sigma_z \neq 0$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \frac{E}{(1+\mu)(1-2\mu)} \begin{bmatrix} 1-\mu & \mu & \mu & 0 & 0 & 0 \\ \mu & 1-\mu & \mu & 0 & 0 & 0 \\ \mu & \mu & 1-\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\mu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\mu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\mu}{2} \end{bmatrix} \begin{Bmatrix} e_x \\ e_y \\ e_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}$$

Here also $e_z = 0$, $\gamma_{yz} = 0$, $\gamma_{zx} = 0$. Neglect those rows

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\mu)(1-2\mu)} \begin{bmatrix} 1-\mu & \mu & 0 \\ \mu & 1-\mu & 0 \\ 0 & 0 & \frac{1-2\mu}{2} \end{bmatrix} \begin{Bmatrix} e_x \\ e_y \\ \gamma_{xy} \end{Bmatrix}$$

$$[D] = \frac{E}{(1+\mu)(1-2\mu)} \begin{bmatrix} 1-\mu & \mu & 0 \\ \mu & 1-\mu & 0 \\ 0 & 0 & \frac{1-2\mu}{2} \end{bmatrix}$$